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Paramagnetism in colour superconductivity and compact stars

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Abstract

It is quite plausible that colour superconductivity occurs in the inner regions of neutron stars. At the same time, it is known that strong magnetic fields exist in the interior of these compact objects. In this paper we discuss some important effects that can occur in the colour superconducting core of compact stars due to the presence of the stars' magnetic field. In particular, we consider the modification of the gluon dynamics for a colour superconductor with three massless quark flavours in the presence of an external magnetic field. We show that the long-range component of the external magnetic field that penetrates the colour-flavour locked phase produces an instability for field values larger than the charged gluons' Meissner mass. As a consequence, the ground state is restructured forming a vortex state characterized by the condensation of charged gluons and the creation of magnetic flux tubes. In the vortex state the magnetic field outside the flux tubes is equal to the applied one, while inside the tubes its strength increases by an amount that depends on the amplitude of the gluon condensate. This paramagnetic behaviour of the colour superconductor can be relevant for the physics of compact stars.

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1. Introduction

In the realm of high density and low temperature QCD baryons get so squeezed that they start to overlap, thereby erasing any vestige of structure. Since in that situation the quarks get very close to each other, their interactions become weak due to asymptotic freedom. At densities of the order of ten times the nuclear density ($\sim 2-4\times 10^{15}$ g cm^{-3}) the weakly interacting quarks can exist out of confinement. In nature the combination of such densities and relatively low temperatures exist in the core of neutron stars, which are the remnant of supernova explosions. It has been predicted on purely theoretical grounds that if the remnant of a supernova explosion has sufficiently high density, it could lead to the formation of a quark star [1]. This compact object would be something in between a neutron star and a black hole.

At very low temperatures a finite density of fermions will fill out all the lowest available energy states up to the Fermi energy. Fermions in the Fermi surface have the same energy, but different momentum. If there were no attractive interaction between the fermions sitting on the Fermi surface, this realization will be the system final state. However, an arbitrarily weak attractive interaction among those fermions will render the existing ground state unstable favouring the formation of fermion–fermion pairs. This restructuring of the ground state is the basis of the phenomena of superconductivity and superfluidity.

In QCD the fundamental interaction between two quarks is attractive. Hence, at very large densities the arbitrarily weak interaction between the asymptotically free quarks on the Fermi surface will do the trick of restructuring the ground state through the formation of Cooper pairs of quarks with opposite spin and momentum [2]. Because the quarks carry 'colour' charge, the quark–quark pairs will carry nonzero colour charge too, thus the name of colour superconductivity.

On the other hand, it is well known that strong magnetic fields, as large as $B \sim 10^{12} - 10^{14}$ G, exist in the surface of neutron stars [3], while in magnetars they are in the range $B \sim 10^{14} - 10^{15}$ G, and perhaps as high as 10^{16} G [4]. It is presumed from the virial theorem [5] that the interior field in neutron stars could be as high as $10^{18} - 10^{19}$ G. If quark stars are self-bound rather than gravitational-bound objects, the previous upper limit that has been obtained by comparing the magnetic and gravitational energies could go even higher. Thus, investigating the effect of strong magnetic fields in colour superconductivity is of interest for the study of compact stars in astrophysics.

To consider the magnetic field interaction with the particles immersed in the colour superconductor (CS) we should have in mind that there the quark–quark pairs carry both colour and electric charges. Hence a CS is also an electric superconductor. One might think that because of this, a magnetic field cannot penetrate the colour superconductor. But in this complex medium something qualitatively new takes place; the electromagnetic field mixes up with one of the gluons to form a new 'electromagnetic' field (called in the literature a 'rotated' electromagnetic field, where the 'rotation' takes place here in an inner space) [6]. This 'rotated' electromagnetic field $\widetilde{A}$ remains long range within the superconductor, because the quark pairs are all neutral with respect to the corresponding 'rotated' electromagnetic charge $\widetilde{Q}$. Therefore, there is no Meissner effect for the corresponding rotated magnetic field $\widetilde{H}$.

In recent works [7, 8] we showed that the properties of the CS can be substantially transformed by the penetrating $\widetilde{H}$ field. First, the pairing of (rotated) electrically charged quarks is reinforced by the field [7]. Pairs of this kind have bounding energies which depend on the magnetic-field strength and are bigger than those existing at zero field. Second, the symmetry of the superconducting phase is changed, because now the magnetic field differentiates the condensates which get contributions from pairs formed by $\widetilde{Q}$-charged quarks from those that only get contributions from pairs formed by $\widetilde{Q}$-neutral quarks [7]. Due to the symmetry change, the low-energy physics of the superconductor is also changed. This last effect can have practical implications for astrophysics since all the transport properties of the star are basically managed by the low-energy physics of the phase. In particular, the cooling of the star is determined by the particles with the lowest energy; so a star with a core of quark matter and a sufficiently large magnetic field can have a distinctive cooling process. This is a point that deserves to be investigated in more detail. Finally, the magnetic field can also influence the gluon dynamics [8]. At field strengths comparable to the charged gluon Meissner mass an inhomogeneous condensate of $\widetilde{Q}$-charged gluons is formed [8]. The gluon condensate anti-screens the magnetic field due to the anomalous magnetic moment of these spin-1 particles. Because of the anti-screening, this condensate does not give a mass to the $\widetilde{Q}$

photon, but instead amplifies the applied rotated magnetic field. This means that in the CS a sort of anti-Meissner effect takes place. This last effect can be also of interest for astrophysics since once the core of a compact star becomes colour superconducting, its internal magnetic field can be boosted to values higher than those found in neutron stars with cores of nuclear matter. This effect could open a new window to differentiate a neutron star made up entirely of nuclear matter from one with a quark matter core. In this paper we will discuss the mechanism that generates this kind of paramagnetism in colour superconductivity.

2. Gluon instability at $\widetilde{H} \geqslant \widetilde{H}_C$

There is a similarity between the electroweak symmetry-broken phase and the colour superconducting phase of QCD. In the first model, the Higgs condensate although blocks the penetration of the hypermagnetic field, allows a combination of the hyperfield and one of the weak isospin fields to penetrate the symmetry-broken medium. As known, the corresponding penetrating field is the electromagnetic field A, which is the only remaining long-range field in that phase. The $W^{\pm}$ bosons, although neutral with respect to the hypercharge, acquire electromagnetic charges in the new phase. In the CS, the role of the electromagnetic field is played by the linear combination $\widetilde{A}_\mu = \cos\theta A_\mu + \sin\theta G^8_\mu$ of the photon A_μ and the gluon G^8_μ fields. Even though gluons are neutral with respect to the conventional electromagnetism, in the colour superconducting phase they acquire $\widetilde{Q}$ charges:

G^1_μ	G^2_μ	G^3_μ	G^+_μ	G^-_μ	I^+_μ	I^-_μ	$\widetilde{G}^8_\mu$
0	0	0	1	−1	1	−1	0

, (1)

given in units of $\widetilde{e} = e\cos\theta$. The $\widetilde{Q}$-charged fields in (1) correspond to the combinations $G^{\pm}_\mu \equiv \frac{1}{\sqrt{2}}\left[G^4_\mu \mp \mathrm{i}G^5_\mu\right]$ and $I^{\pm}_\mu \equiv \frac{1}{\sqrt{2}}\left[G^6_\mu \mp \mathrm{i}G^7_\mu\right]$.

Taking into account the Schwinger energy spectrum of a charged particle of spin s, charge e, gyromagnetic ratio g, and mass m in a magnetic field H,

$$E_n^2 = (2n+1)eH - ge\mathbf{H}\cdot\mathbf{s} + m^2, \tag{2}$$

we see that for spin-1 particles (i.e. $g = 2$ and spin projection $-1, 0, +1$) $E^2 < 0$ for strong enough magnetic fields ($H > H_{\mathrm{cr}} = m^2/e$). Therefore, when the field surpasses the critical value H_{cr}, one of the modes of the charged gauge field becomes tachyonic (this is the well-known 'zero-mode problem' found in the presence of a magnetic field for Yang–Mills fields [9], for the $W^{\pm}_\mu$ bosons in the electroweak theory [10, 11], and even for higher spin fields in the context of string theory [12]).

Similarly to other spin-1 theories with magnetic instabilities [9–11], the charged gluons in the magnetized CS suffer of instabilities for $\widetilde{H} > \widetilde{H}_C = m_M^2/\widetilde{e}$, where m_M is the charged gluon Meissner mass. To remove the magnetically induced instabilities, a vortex ground state is formed [8]. This vortex state is characterized by the condensation of charged gluons and the creation of 'rotated' magnetic flux tubes.

3. Gluon vortex condensate and paramagnetism

Since at densities high enough to neglect the strange quark mass, the ground state of three-flavour quark matter corresponds to the colour-flavour locked (CFL) phase [6], we will focus our analysis into this phase, although the conclusions can be easily extrapolated to other phases, as the 2SC phase, for example.

Above the critical field $\left(\widetilde{H}_C = m_M^2/\widetilde{e}\right)$ oriented along the Z-axis, the mass mode that becomes tachyonic, corresponds to a charged field eigenvector of amplitude G in the $(1, \mathrm{i})$

spatial (x, y)-direction for G^- (G^* in the $(1, -i)$ direction for G^+). Without loss of generality, we are only considering in this analysis the set of charged fields $G^{\pm}_{\mu}$. To remove the tachyonic mode, the ground state is restructured through the formation of a gauge field condensate G, as well as an induced magnetic field $\widetilde{\mathbf{B}} = \nabla \times \widetilde{\mathbf{A}}$ that is originated due to the backreaction of the G condensate on the rotated electromagnetic field.

The condensate solutions can be found by minimizing with respect to G and $\widetilde{B}$ the Gibbs free energy density $\mathcal{G}_c = \mathcal{F} - \widetilde{H}\widetilde{B}$ ($\mathcal{F}$ is the free energy density),

$$\mathcal{G}_c = \mathcal{F}_{n0} - 2G^{\dagger}\widetilde{\Pi}^2 G - 2\big(2\widetilde{e}\widetilde{B} - m_M^2\big)|G|^2 + 2g^2|G|^4 + \tfrac{1}{2}\widetilde{B}^2 - \widetilde{H}\widetilde{B}. \qquad (3)$$

In (3) $\mathcal{F}_{n0}$ is the system free energy density in the normal-CFL phase ($G = 0$) at zero applied field. Using (3) the minimum equations at $\widetilde{H} \sim \widetilde{H}_C$ for the condensate G and induced field $\widetilde{B}$ respectively are

$$-\widetilde{\Pi}^2 G - \big(2\widetilde{e}\widetilde{B} - m_M^2\big)G = 0, \qquad (4)$$

$$2\widetilde{e}|G|^2 - \widetilde{B} + \widetilde{H} = 0. \qquad (5)$$

Identifying G with the complex order parameter, equations (4)–(5) become analogous to the Ginzburg–Landau equations for a conventional superconductor except for the $\widetilde{B}$ contribution in the second term in (4) and the sign of the first term in (5). The origin of both terms can be traced back to the anomalous magnetic moment term $4\widetilde{e}\widetilde{B}|G|^2$ in the Gibbs free energy density (3). Note that because of the different sign in the first term of (5), contrary to what occurs in conventional superconductivity, the resultant field $\widetilde{B}$ is stronger than the applied field $\widetilde{H}$. Thus, when a gluon condensate develops, the magnetic field will be anti-screened and the CS will behave as a paramagnet. The anti-screening of a magnetic field has been also found in the context of the electroweak theory for magnetic fields $H \geqslant M_W^2/e \sim 10^{24} G$ [11]. Just as in the electroweak case, the anti-screening in the CS is a direct consequence of the asymptotic freedom of the underlying theory [11, 13].

We should highlight that the gluon condensate discussed in this work is not the only charged spin-1 condensate generated in a theory with a large fermion density. As known [14], a spin-1 condensate of $W^{\pm}$-bosons can be originated at sufficiently high fermion density in the context of the electroweak theory at zero magnetic field. However, the physical implications of the gluon condensate induced by the magnetic field in the CS are fundamentally different from those associated with the homogeneous $W^{\pm}$-boson condensate of the dense electroweak theory [14]. The gluon vortices in the magnetized CS boost the applied field, leaving the $\widetilde{Q}$ photon massless and thereby preserving the $\widetilde{U}_{\mathrm{em}}(1)$ symmetry. On the other hand, the $W^{\pm}$-boson condensate breaks the $U_{\mathrm{em}}(1)$ symmetry turning the electroweak system in an electromagnetic superconductor [15].

The explicit solution of (4) with vanishing conditions at $x \to \pm\infty$, can be found following Abrikosov's approach [16] to type II metal superconductivity for the limit situation when the applied field is near the critical value H_{c2}. In our case we find

$$G_k = \exp[-\mathrm{i}ky]\exp\left[-\frac{(x - x_k)^2}{2\xi^2}\right], \qquad (6)$$

where $k \equiv k_y$. From the experience with conventional type II superconductivity [17] it is known that to minimize the energy the inhomogeneous condensate solutions secure a periodic lattice structure. Then, putting on periodicity in the y-direction with period $\Delta y = b$ restricts the values of k to a discrete set $k = 2\pi n/b, n = 1, 2, \ldots$. This condition implies that we have an infinite set of discrete solutions that superpose to form the general solution $G(x, y) = \sum C_n G_n$. This superposition of all the Gaussian solutions centred at different x_n

constitutes the vortex state that removes the instability in the whole space. On the other hand, the discrete values of k imply periodicity in x, since the Gaussian solutions G_n are located at $x_n = \frac{k_n \widetilde{\Phi}_0}{2\pi \widetilde{H}_C} = \frac{n\widetilde{\Phi}_0}{b\widetilde{H}_C}$, with $\widetilde{\Phi}_0 \equiv 2\pi/\widetilde{e}$. Assuming then that all G_n enter with equal weight, the periodicity length in the x-direction is $\Delta x = \frac{\widetilde{\Phi}_0}{b\widetilde{H}_C}$. Therefore, the magnetic flux through each periodicity cell in the vortex lattice is quantized $\widetilde{H}_C \Delta x \Delta y = \widetilde{\Phi}_0$, with $\widetilde{\Phi}_0$ being the flux quantum per unit vortex cell. In this semi-qualitative analysis we considered Abrikosov's ansatz of a rectangular lattice (i.e. all the coefficients C_n being equal). For the rectangular lattice, the area of the unit cell is $A = \Delta x \Delta y = \widetilde{\Phi}_0/\widetilde{H}_C$, so decreasing with $\widetilde{H}$.

4. Conclusions and final remarks

In conclusion, at low $\widetilde{H}$ field the CS behaves as an insulator that can be penetrated by the $\widetilde{H}$ field. When the $\widetilde{H}$ field reaches the critical value $\widetilde{H}_C = m_M^2/\widetilde{e}$, the condensation of charged gluons is triggered inducing the formation of a lattice of magnetic flux tubes and breaking the translational and remaining rotational symmetries. Contrary to the situation in conventional type II superconductors, where the applied field only penetrates through the flux tubes and with a smaller strength, the vortex state in the CS has the peculiarity that outside the flux tube the applied field $\widetilde{H}$ totally penetrates the sample, while inside the tubes the magnetic field becomes larger than $\widetilde{H}$. This anti-screening behaviour is similar to that of the electroweak system at high magnetic field [11]. Note that as the $\widetilde{Q}$ photons remain massless in the presence of the condensate G, the $\widetilde{U}(1)_{\rm em}$ symmetry remains unbroken.

A rough estimate of the critical field that produces the magnetic instability at the scale of baryon densities typical of neutron-star cores ($\mu \simeq 200 - 400$ MeV, $\alpha_s(\mu) \simeq 1/3$) gives $\widetilde{H}_C \simeq 9.5 \times 10^{16}$ G $- 3.8 \times 10^{17}$ G. Although these are significantly high magnetic fields, they cannot be ruled out as acceptable values for the neutron-star core.

At present, there is a lot of activity among the physics community trying to find ways to differentiate a neutron star made up entirely of nuclear matter from that with a quark colour superconducting core. Some guiding ideas in this direction have been to link the phase of the star's core to measurable properties of the star as its radio-mass ratio, its cooling process, and its rotational and vibrational properties. In this regard, the result that we are reporting on the increase of the star's magnetic field by the realization of colour superconductivity in its core can also serve to that goal, and deserves further investigations.

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